Indian Statistical Institute, Bangalore

B. Math.(Hons.) III Year, Second Semester

Mid-Sem Examination

Combinatorics and Graph Theory

Time: 3 hours March 1, 2010 Instructor: N.S.N.Sastry

Maximum Marks 100

1. Define a Hadamard design. Show that a Hadamard design admits a unique extension. [10]

2. Define a $t-$ transitive permutation group. Find the largest integer t for which the alternating group A_n acting on $\{1, \cdots, n\}$ is $t-$ transitive. [10]

3. Define a projective plane of order n. Show that a projective plane of order 2 exists and is unique. [10]

4. For $t \in \{0, 1, \cdots, n-1\}$, let $\mathbb{P}_t^n(q)$ denote the set of all subspaces of $\mathbb{F}_q^{n+1}$ of dimension $t+1$.

   a) Show that $X = (\mathbb{P}_0^n(q), \mathbb{P}_{n-1}^n(q))$ is a symmetric design. Calculate its parameters

   b) If X as in (a) is an extendable design, then show that $n = 2$ and $q \in \{2, 4\}$.

   [10+12]

5. Let G be the group of all $\mathbb{F}_q-$ vector space automorphisms of $\mathbb{F}_q^{n+1}$. Consider the action of G on $\mathbb{P}_t^n(q)$ induced by the natural action of G on $\mathbb{F}_q^{n+1}$ $(0 \leq t \leq h)$. Determine the number of $G-$ orbits for the action of G on $\mathbb{P}_t^n(q) \times \mathbb{P}_t^n(q)$. [12]

6. If $\mathbb{D} = (X, \mathbb{B})$ is a $t-(v, k, \lambda)$ design with $\mid \mathbb{B} \mid > 1$ and $t > 2$, then show that $\mid X \mid < \mid \mathbb{B} \mid$.

   [8]

7. Show that every nontrivial automorphism of a $t-(v, k, \lambda)$ design, $t \geq 2$ and $k > \lambda$, moves at least one block. [8]

8. Let $\mathbb{D} = (X, \mathbb{B})$ be an extension of a symmetric $2-(v, k, \lambda)$ design. Let $B_0 \in \mathbb{B}$ and $\mathbb{B}^1$ be the set of all elements of $\mathbb{B}$ which are disjoint from B_0. Show that $(X \backslash B_0, \mathbb{B}^1)$ is a $2-$design. Calculate its parameters.

   [10]

9. For each positive integer n and prime p, show that there exists a polynomial $f(x) \in \mathbb{F}_p[X]$ of degree n which is irreducible over $\mathbb{F}_p$. Deduce the existance of a field of order P^n for each prime p and each integer $n > 0$. [10]

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